An effective integration of subdivision surfaces in a hybrid computer-aided design system

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Subdivision surfaces

- 👍 Widely supported in nearly all modeling programs
- 👍 Advantages: flexibility for arbitrary topology + superset of NURBS "standard"
- 👍 A lot of theoretical study and many proposed algorithms *potentially* useful in CAD, such as surface fitting, reverse engineering, curve lofting
- 👎 Their presence in CAD is still negligible due to:
  - lack of closed-form representation (difficulty of integration into the system)
  - quality and regularity issues around extraordinary points
- ▶ But CAD end-users ask for them!

Project NIIT4CAD

The main **objective** of the European project NIIT4CAD is an effective integration of Catmull-Clark subdivision surfaces in a CAD system, which means that:

- the desired accuracy is achieved
- all the functionalities of the CAD system are inherited

Approach:

- Suitable local correction of Catmull-Clark surface, capable of guaranteeing the required analytic accuracy
- Seamless integration into the modeling workflow, exploiting an extensible geometric kernel and the possibility of handling a hybrid boundary representation

CAD system paradigm

Our integration approach relies on a CAD system based on an extensible geometric kernel, which means:

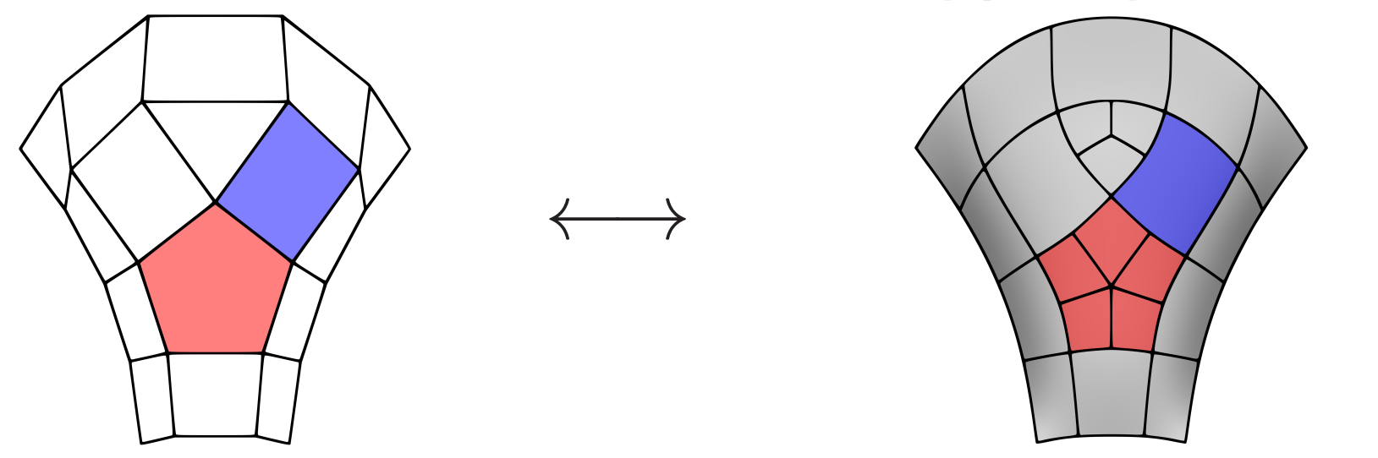
- **Parametrization and evaluation paradigm**: any geometric entity, represented in parametric form, can be integrated by simply implementing the related evaluation algorithm, that acts as an interface with the entire system. In this way, all the functionality of the CAD system are automatically inherited by any newly introduced type of representation
- Extended **boundary representation** (B-rep), which allows non-solid model types, so as to integrate surface modeling into a solid modeling environment (hybrid CAD system)
- **Hybrid description of the geometry** in the B-rep, that is allowing for the coexistence of different forms of geometric representation in the same model

Integration through the Subd B-rep

A **Subd B-rep** represents a B-rep geometric model in which:

- the B-rep topology is inferred from the subdivision control mesh
- the geometry of each B-rep face is described as a subdivision surface patch associated with a rectangular parametric domain

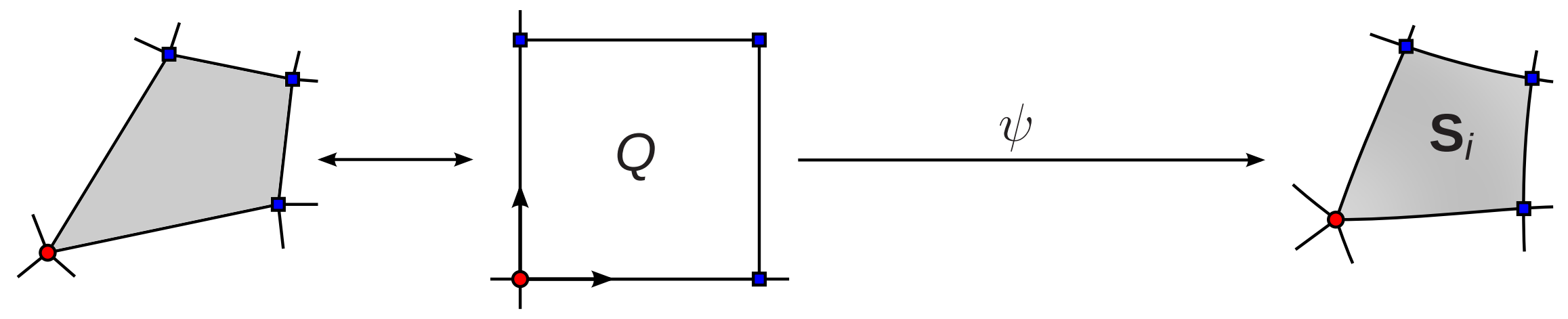
	Control mesh	⟷	Subd B-rep
• Topology:	1 quad face	⟷	1 face
	1 n-sided face	⟷	n (quad) faces



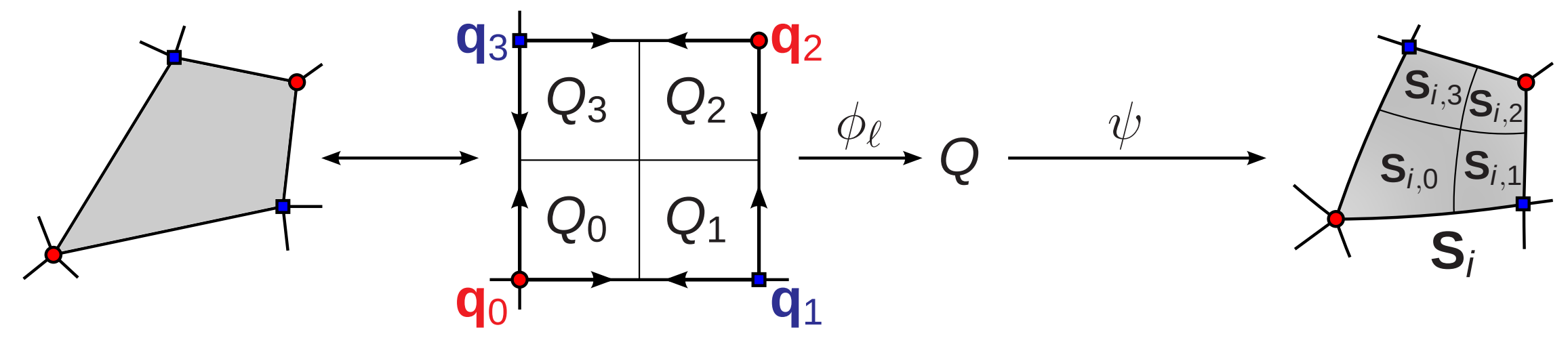
The Subd B-rep maintains an intuitive association (1-1 almost everywhere) between the control mesh faces and the B-rep faces

• Geometry:	1 face	⟷	1 parametric patch $\mathbf{S}_i$ on $Q := [0,1]^2$

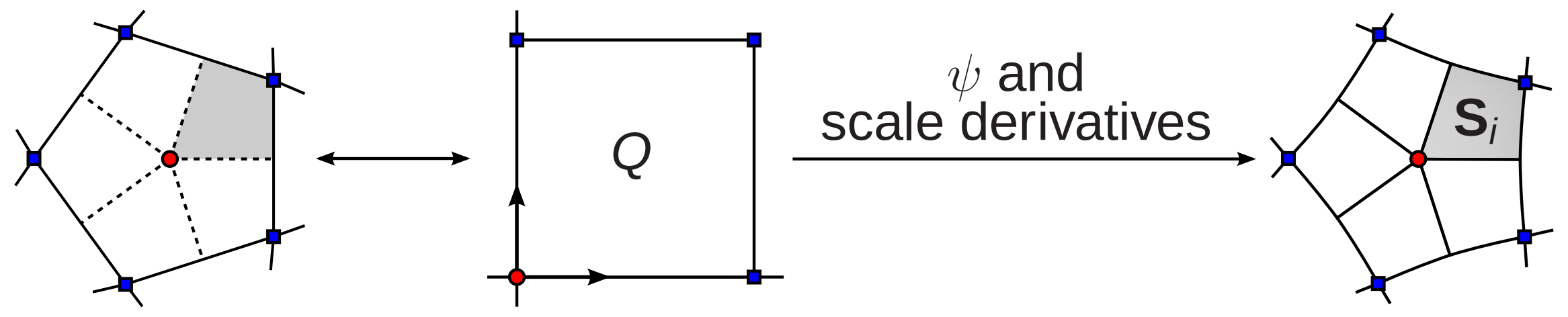
(a) Quad face with a single extraordinary vertex (e.v.)



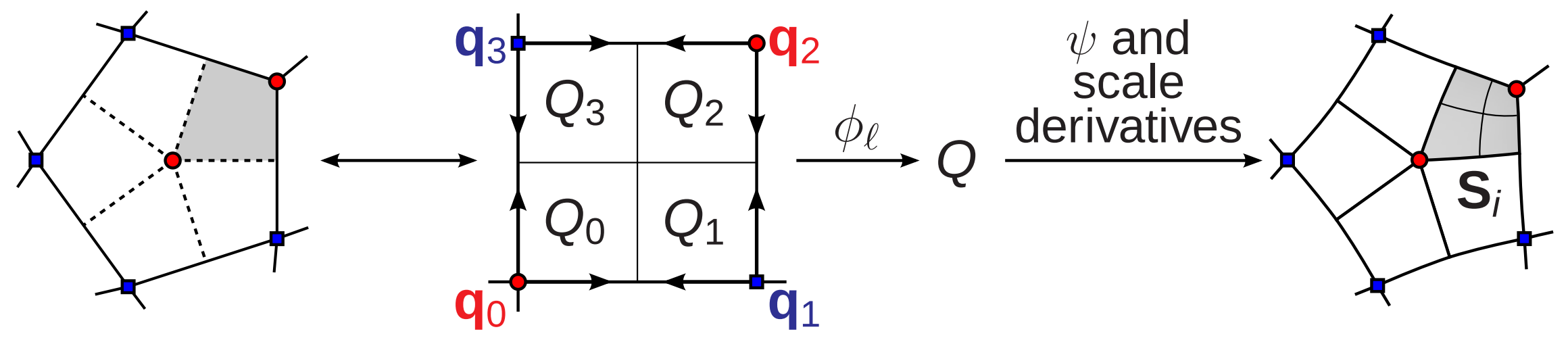
(b) Quad face with more than one e.v.



(c.1) Non-quad face without e.v.



(c.2) Non-quad face with at least one e.v.



where:

ψ Catmull-Clark evaluator [4]

$\phi_\ell : Q_\ell \longrightarrow Q, \quad \phi_\ell := \sigma_2 \circ \rho^{-1}_{\ell\frac{\pi}{2}} \circ \tau_{\mathbf{q}_\ell - \mathbf{q}_0}$

σ_h scaling by a factor h

ρ_a^{-1} c.w. rotation around the origin of an angle a

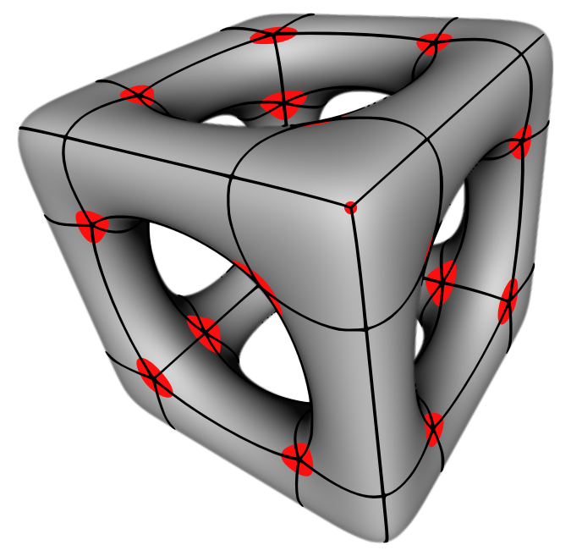
$\tau_{\mathbf{v}}$ translation of a vector $\mathbf{v}$

Local correction around extraordinary points

We aim at maintaining the nice shape and B-spline nature of Catmull-Clark surfaces in the widest possible area, while improving the analytic properties in the smallest neighborhood of e.v.

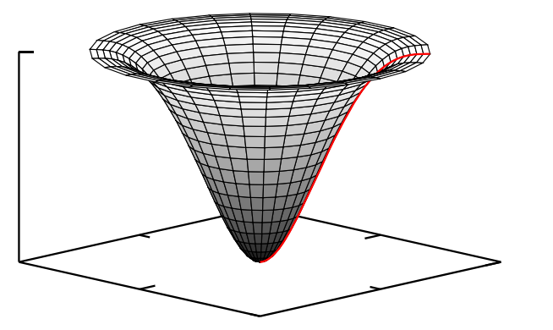
Polynomial blending: the Catmull-Clark surface is locally blended with a suitable polynomial surface [3, 5]

$$\underbrace{\boxed{\mathbf{S}_i^*}}_{\text{blended surface}} := \underbrace{\boxed{w}}_{\text{weight function}} \overbrace{\boxed{\mathbf{S}_i}}^{\text{Catmull-Clark surface}} + (1-w)\underbrace{\boxed{\mathbf{P}}}_{\text{approximating polynomial}}$$



blending regions

- w s.t. C^2-transition between $\mathbf{S}_i^*$ and $\mathbf{S}_i$



Our approach generates a parametric surface evaluable at arbitrary points

- $\mathbf{S}_i, \mathbf{P}, w$ must be parameterized over a common domain
- We represent the characteristic map of the subdivision scheme for valence n as a parametric multipatch surface $\psi_{\mathbf{K}_i^{[n]}} : Q \longrightarrow \mathbf{K}_i^{[n]}, i = 0, \ldots, n$ (n sectors with rotational symmetry)
- Star-shaped transformation

  $\kappa_i := \rho_{i\frac{2\pi}{n}} \circ \psi_{\mathbf{K}_0^{[n]}} \circ \sigma_h \circ \phi$

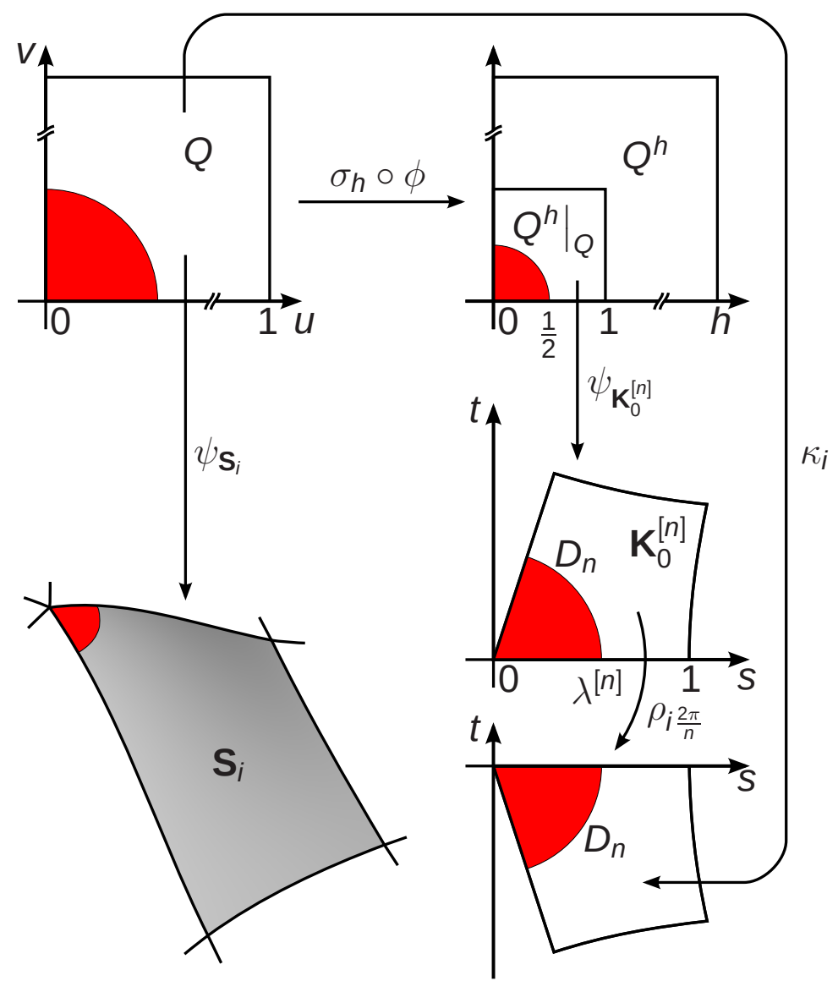
  where:

  $$\phi(u,v) := \begin{cases} (u,v) & \text{patch type (a), (c.1)} \\ \phi_\ell(u,v) & \text{patch type (b), (c.2)} \end{cases}$$

  σ_h scaling of $h = 4(\text{a}), 2(\text{b}), 2(\text{c.1}), 1(\text{c.2})$

  $\psi_{\mathbf{K}_0^{[n]}} : Q^h\big|_Q \longrightarrow \mathbf{K}_0^{[n]}, \quad Q^h := [0,h]^2$

  $\rho_{i\frac{2\pi}{n}}$ c.c.w. rotation of angle $i\frac{2\pi}{n}$



- Star-shaped domain

  $$K_n := \bigcup_{i=0}^{n-1} \left\{ (s,t) := \kappa_i(u,v) \;\middle|\; (u,v) \in Q \text{ and } \sigma_h(\phi(u,v)) \in Q \right\}$$

- Blending region

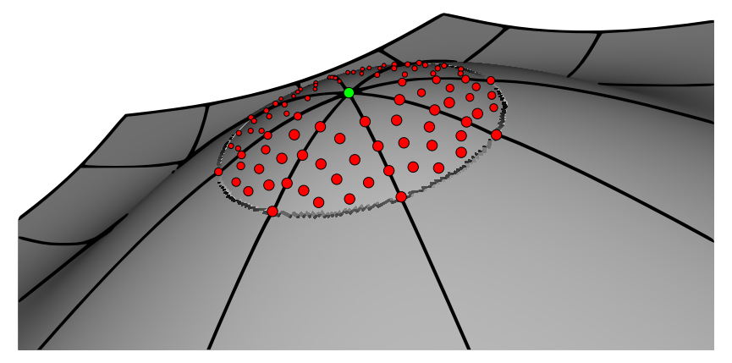
  $$D_n := \left\{ (s,t) \in K_n \;\middle|\; \|(s,t)\|_2 \leqslant \lambda^{[n]} \right\}$$

  where $\lambda^{[n]}$ = subdominant eigenvalue of the subdivision matrix

- Blended surface

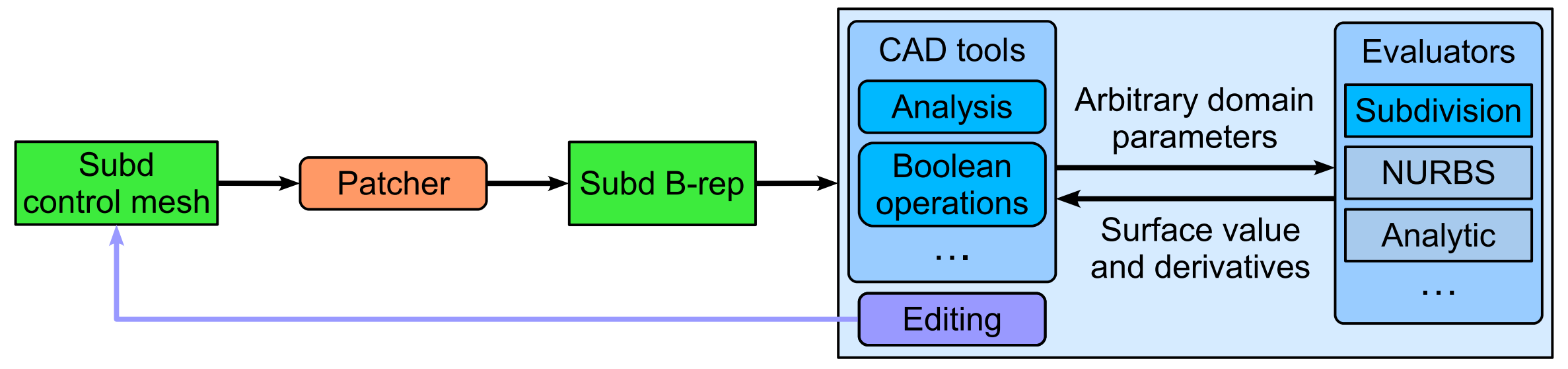
  $$\mathbf{S}_i^*(u,v) := \begin{cases} w(s,t)\mathbf{S}_i(u,v) + (1-w(s,t))\mathbf{P}(s,t) & (s,t) \in D_n \\ \mathbf{S}_i(u,v) & \text{elsewhere} \end{cases}$$

The **approximating polynomial P** is determined by least square fitting inside the blending region with interpolation of the e.v.

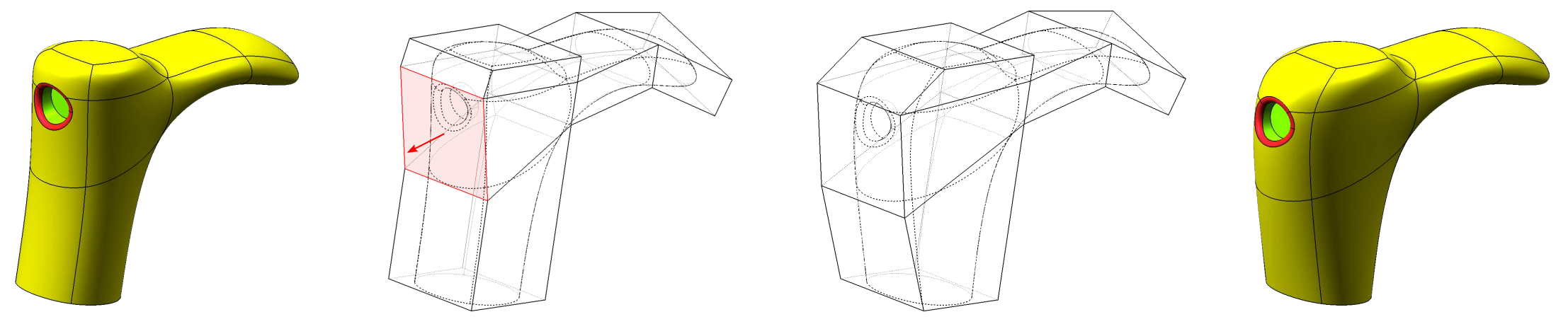


Practical modeling

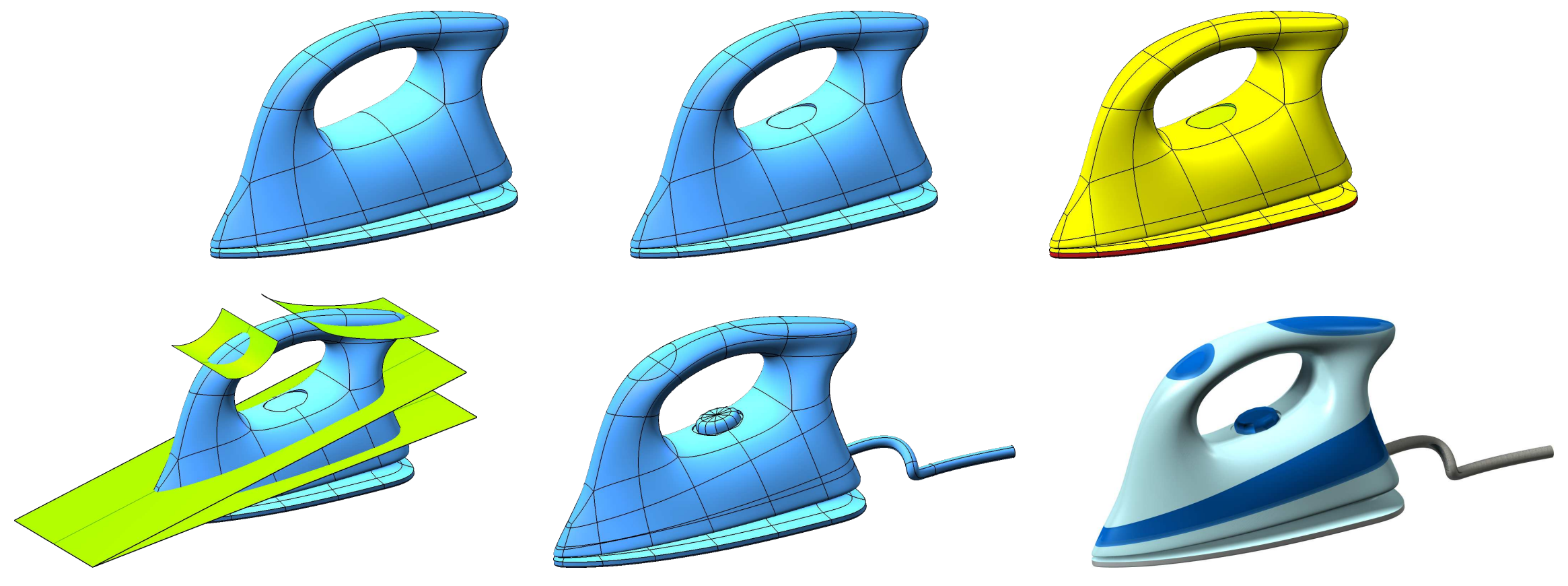
- Workflow for the creation and editing of a Subd B-rep (applies to those faces of the hybrid B-rep model that are of *subdivision* type)

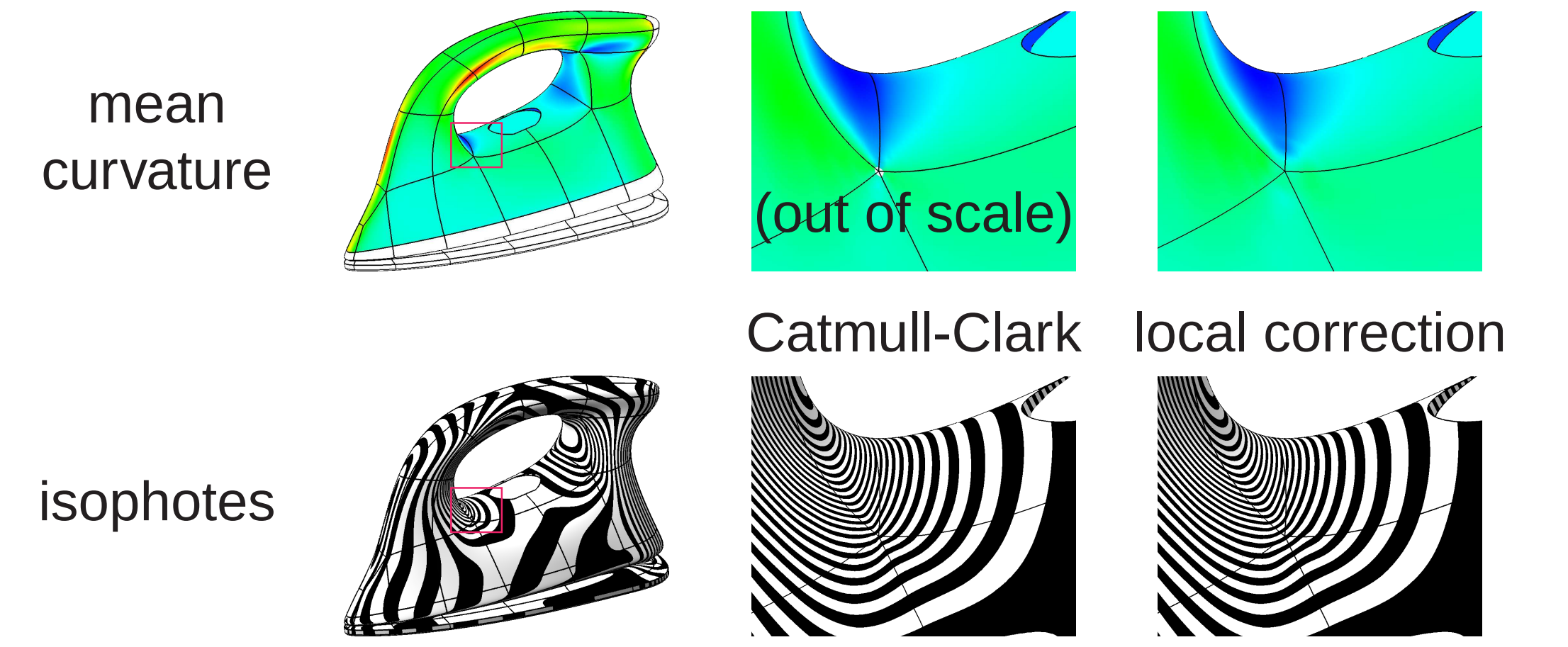


- Operations of solid composition generate a B-rep whose faces can have hybrid nature (e.g. NURBS + subdivision) and are editable while maintaining this feature



- Hybrid model and quality assessment w.r.t. Catmull-Clark surface





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Eurostars project New Interactive and Innovative Technologies for CAD (2010–2013)

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